


Online Workshop on

Stochastic Analysis and Hermite Sobolev Spaces

21–26 June 2021

Aims and scope

The goal of the workshop is to expose research scholars, post doctoral fellows and young faculty to a few techniques that have been developed recently in the context of stochastic processes taking values in the Hermite-Sobolev spaces. This includes the following topics:

- Monotonicity inequality and uniqueness of solutions for SPDEs
- Translation invariance of solutions to SPDEs
- Connections to Geometry, invariant submanifolds
- Applications to Finance and Machine learning
- Applications to Stochastic Analysis & PDEs

While we shall assume basic knowledge of stochastic calculus, we shall also develop in some detail the necessary topics in distribution theory and stochastic integration in finite dimensions and later on, in Hilbert spaces.

The Hermite-Sobolev spaces describe the countably Hilbertian Nuclear topology of the Schwartz space of rapidly decreasing smooth functions; their duals provide a convenient framework for the study of Stochastic PDEs, including Stochastic Differential Equations in finite dimensions.

Recent developments suggest a broader range of applications for these techniques. This is also a motivation for the workshop, which will include lectures by experts in stochastic analysis with the goal of formulating and solving new problems in the framework of Hermite-Sobolev spaces.

Confirmed speakers include

- Eduardo Abi Jaber (University of Paris, France)
- Vladimir Bogachev (Lomonosov Moscow State University & Higher School of Economics, Russia)
- Damir Filipović (EPF Lausanne and Swiss Finance Institute, Switzerland)
- Martin Grothaus (University of Kaiserslautern, Germany)
- Rajat Subhra Hazra (Indian Statistical Institute, Kolkata & Leiden University, Netherlands)
- Bernt Øksendal (University of Oslo, Norway)
- Szymon Peszat (Jagiellonian University Cracow, Poland)
- Michael Röckner (University of Bielefeld, Germany)
- Sivaguru S. Sritharan (Applied Optimization, Inc. USA)
- Josef Teichmann (ETH Zurich, Switzerland)
- Sundaram Thangavelu (Indian Institute of Science, Bangalore, India)

Organizers

- Rajeev Bhaskaran (Indian Statistical Institute, Bangalore Centre, India)
- Suprio Bhar (Indian Institute of Technology Kanpur, India)
- Stefan Tappe (Albert Ludwig University of Freiburg, Germany)

Registration

The workshop has no fees. Registration should be done by e-mail to stahss2021@stochastik.uni-freiburg.de.

For more information see https://www.stoch-ana-hermite-sobolev.uni-freiburg.de/

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Albert-Ludwigs-Universität Freiburg, Indian Institute of Technology Kanpur, Indian Statistical Institute Bangalore

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Program

Monday, 21 June 2021

10:30 – 11:30 Central European Time (2:00 pm - 3:00 pm Indian time)
Rajeev Bhaskaran
Introduction and Motivation for Stochastic Calculus in Hermite Sobolev spaces

11:30 – 12:30 Central European Time (3:00 pm - 4:00 pm Indian time)
Suprio Bhar
A Short Introduction to Distribution Theory

13:30 – 14:30 Central European Time (5:00 pm - 6:00 pm Indian time)
Stefan Tappe
A Short Introduction to Stochastic Integration in Hilbert Spaces

14:30 – 15:30 Central European Time (6:00 pm - 7:00 pm Indian time)
Sundaram Thangavelu
On fractional powers of the Hermite operator and associated Sobolev spaces

Tuesday, 22 June 2021

10:30 – 11:30 Central European Time (2:00 pm - 3:00 pm Indian time)
Suprio Bhar
A Short Introduction to Distribution Theory

11:30 – 12:30 Central European Time (3:00 pm - 4:00 pm Indian time)
Stefan Tappe
A Short Introduction to Stochastic Integration in Hilbert Spaces

13:30 – 14:30 Central European Time (5:00 pm - 6:00 pm Indian time)
Sundaram Thangavelu
On fractional powers of the Hermite operator and associated Sobolev spaces

14:30 – 15:30 Central European Time (6:00 pm - 7:00 pm Indian time)
Damir Filipović
Machine Learning With Kernels for Portfolio Valuation and Risk Management

Wednesday, 23 June 2021

10:30 – 11:30 Central European Time (2:00 pm - 3:00 pm Indian time)
Suprio Bhar
A Short Introduction to Distribution Theory

11:30 – 12:30 Central European Time (3:00 pm - 4:00 pm Indian time)
Rajeev Bhaskaran
From SDE's to SPDE's

13:30 – 14:30 Central European Time (5:00 pm - 6:00 pm Indian time)
Rajat Subhra Hazra
Scaling limit of some random interface models

14:30 – 15:30 Central European Time (6:00 pm - 7:00 pm Indian time)
Josef Teichmann
Gaussian processes, Signatures and Kernelizations

Albert-Ludwigs-Universität Freiburg, Indian Institute of Technology Kanpur, Indian Statistical Institute Bangalore

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Program

Thursday, 24 June 2021

10:30 – 11:30 Central European Time (2:00 pm - 3:00 pm Indian time)
Michael Röckner
Nonlinear Fokker-Planck equations with measures as initial data and McKean-Vlasov equations

11:30 – 12:30 Central European Time (3:00 pm - 4:00 pm Indian time)
Stefan Tappe
Invariant manifolds in Hermite Sobolev spaces

13:30 – 14:30 Central European Time (5:00 pm - 6:00 pm Indian time)
Szymon Peszat
Heat equations with white noise Dirichlet boundary conditions

14:30 – 15:30 Central European Time (6:00 pm - 7:00 pm Indian time)
Sivaguru S. Sritharan
Navier-Stokes Equations: Ergodicity, Large Deviations, Control, Filtering and Malliavin Calculus

Friday, 25 June 2021

10:30 – 11:30 Central European Time (2:00 pm - 3:00 pm Indian time)
Bernt Øksendal
SPDEs with space interactions - a model for optimal control of epidemics

11:30 – 12:30 Central European Time (3:00 pm - 4:00 pm Indian time)
Stefan Tappe
Invariant manifolds in Hermite Sobolev spaces

13:30 – 14:30 Central European Time (5:00 pm - 6:00 pm Indian time)
Martin Grothaus
An improved characterization theorem - its interpretation in terms of Mallivain calculus and applications to SPDEs

14:30 – 15:30 Central European Time (6:00 pm - 7:00 pm Indian time)
Rajeev Bhaskaran
From SDE's to SPDE's

Saturday, 26 June 2021

10:30 – 11:30 Central European Time (2:00 pm - 3:00 pm Indian time)
Vladimir Bogachev
Fractional Sobolev classes on infinite-dimensional spaces

11:30 – 12:30 Central European Time (3:00 pm - 4:00 pm Indian time)
Rajeev Bhaskaran
From SDE's to SPDE's

13:30 – 14:30 Central European Time (5:00 pm - 6:00 pm Indian time)
Suprio Bhar
A class of Stochastic PDEs in $\mathscr{S}'$ driven by Lévy noise

14:30 – 15:30 Central European Time (6:00 pm - 7:00 pm Indian time)
Eduardo Abi Jaber
Linear-Quadratic control of stochastic Volterra equations

Albert-Ludwigs-Universität Freiburg, Indian Institute of Technology Kanpur, Indian Statistical Institute Bangalore

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Titles and Abstracts for the Workshop

Rajeev Bhaskaran (Indian Statistical Institute, Bangalore Centre, India)

Introduction and Motivation for Stochastic Calculus in Hermite Sobolev spaces

Abstract: In this introductory talk, after a quick review of finite dimensional stochastic calculus we try to provide a motivation for extending this calculus to a class of Hilbert spaces of distributions viz. the Hermite Sobolev spaces.

Rajeev Bhaskaran (Indian Statistical Institute, Bangalore Centre, India)

From SDE's to SPDE's

Abstract: In this series of three talks, we will begin with a proof of the Itô formula for translations of tempered distributions by continuous semi-martingales. This allows us to formulate Itô's SDEs as nonlinear SPDEs, albeit with initial conditions that are arbitrary tempered distributions. At the same time, when the coefficients are smooth, we use the associated stochastic flow to define the `adjoint flow' and show that it satisfies a linear SPDE. We obtain stochastic representations of the solutions of Kolmogorov's forward equation. We then prove the `Monotonicity inequality' which leads us to the uniqueness of solutions to the above SPDE's and related stochastic evolution systems. Time permitting, we will discuss a general result on the existence and uniqueness of nonlinear SPDE's with Lipschitz continuous coefficients. These lectures are based on joint work with S. Thangavelu, S. Bhar, B. Sarkar, V. Mandrekar and L. Gawarecki.

Suprio Bhar (Indian Institute of Technology Kanpur, India)

A Short Introduction to Distribution Theory

Abstract: In a series of lectures, we discuss the spaces of distributions, which are continuous linear functionals on specific function spaces. First, we consider "Distributions" and "Distributions with compact support", and look at various operations on such spaces. Later, we discuss in detail the space of "Tempered Distributions", which consists of continuous linear functionals on the space of Schwartz class functions.

Suprio Bhar (Indian Institute of Technology Kanpur, India)

A class of Stochastic PDEs in $\mathscr{S}'$ driven by Lévy noise

Abstract: In this talk, we consider a class of finite dimensional SDEs driven by a Lévy noise and a related class of Stochastic PDEs in the space of Tempered distributions $\mathscr{S}'$, driven by the same Lévy noise. We discuss the existence of such Stochastic PDEs by Itô's formula for translation operators and uniqueness by an application of `Monotonicity inequality'. The `translation invariance' property for the solutions shall also be discussed.

Reference

S. Bhar: An Itō Formula in the Space of Tempered Distributions, Journal of Theoretical Probability 30 (2017), 510-528.

S. Bhar, B. Sarkar: Parametric Family of SDEs Driven by Lévy Noise, Communications on Stochastic Analysis 12 (2018), no. 2, Article 4.

S. Bhar, R. Bhaskaran, B. Sarkar: Stochastic PDEs in S ' for SDEs driven by Lévy noise, Random Oper. Stoch Equ. 28 (2020), no. 3, 217-226.

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Titles and Abstracts for the Workshop

Stefan Tappe (Albert Ludwig University of Freiburg, Germany)

A Short Introduction to Stochastic Integration in Hilbert Spaces

Abstract: In a series of two lectures, we develop the stochastic integral in Hilbert spaces. For this purpose, we review the Bochner integral, martingales in Banach spaces, Gaussian random variables in Hilbert spaces and trace class Wiener processes. Once the stochastic integral is established, we will provide an introduction to infinite dimensional SDEs as well as SPDEs in the framework of the semigroup approach

Stefan Tappe (Albert Ludwig University of Freiburg, Germany)

Invariant manifolds in Hermite Sobolev spaces

Abstract: In a series of two lectures, we deal with finite dimensional invariant manifolds for infinite dimensional stochastic equations. First, we demonstrate how this topic arises in interest rate modeling from mathematical finance, and present some results. Then we will consider a general framework with SPDEs in continuously embedded Hilbert spaces, which covers SPDEs in the framework of the semigroup approach. Our results apply in particular to SPDEs with values in Hermite Sobolev spaces. At this juncture there is an interplay between finite dimensional SDEs and particular types of SPDEs. The results about invariant manifolds for SPDEs in continuously embedded Hilbert spaces are based on joint work with Rajeev Bhaskaran.

Sundaram Thangavelu (Indian Institute of Science, Bangalore, India)

On fractional powers of the Hermite operator and associated Sobolev spaces

Abstract: In these two lectures we describe various ways of defining fractional powers of the Hermite operator $H = -\Delta + |x|^2$ and introduce Hermite Sobolev spaces. Realizing the fractional powers H^{-s}, $s > 0$ as pseudo-differential operators, we obtain explicit formulas for their symbols and estimate their derivatives. If time permits, we also say something about their mapping properties.

Titles and Abstracts of Invited Speakers

Eduardo Abi Jaber (University of Paris, France)

Linear-Quadratic control of stochastic Volterra equations

Abstract: We treat Linear-Quadratic control problems for a class of stochastic Volterra equations of convolution type. These equations are in general neither Markovian nor semimartingales, and include the fractional Brownian motion with Hurst index smaller than $1 = 2$ as a special case. We prove that the value function is of linear quadratic form with a linear optimal feedback control, depending on non-standard infinite dimensional Riccati equations, for which we provide generic existence and uniqueness results. Furthermore, we show that the stochastic Volterra optimization problem can be approximated by conventional finite dimensional Markovian Linear Quadratic problems, which is of crucial importance for numerical implementation. Joint work with Enzo Miller and Huyên Pham.

Reference

E. Abi Jaber, E. Miller, H. Pham: Linear-Quadratic control for a class of stochastic Volterra equations: solvability and approximation, arXiv:1911.01900.

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Titles and Abstracts of Invited Speakers

Vladimir Bogachev (Lomonosov Moscow State University and Higher School of Economics, Russia)

Fractional Sobolev classes on infinite-dimensional spaces

Abstract: We discuss fractional Sobolev classes on infinite-dimensional spaces with probability measures, in particular, some analogs of Besov and Nikolskii-Besov classes and BV classes of functions of bounded variation. In the case of a Gaussian reference measure such classes can be defined by means of the Ornstein-Uhlenbeck semigroup or Chebyshev-Hermite expansions, however, there are other interesting options based on certain "nonlinear integration by parts formulas". There are also reasonable similar constructions for more general probability measures.

Damir Filipović (EPF Lausanne and Swiss Finance Institute, Switzerland)

Machine Learning With Kernels for Portfolio Valuation and Risk Management

Abstract: We introduce a simulation method for dynamic portfolio valuation and risk management building on machine learning with kernels. We learn the dynamic value process of a portfolio from a finite sample of its cumulative cash flow. The learned value process is given in closed form thanks to a suitable choice of the kernel. We show asymptotic consistency and derive finite sample error bounds under conditions that are suitable for finance applications. Numerical experiments show good results in large dimensions for a moderate training sample size.

Reference

L. Boudabsa, D. Filipović: Machine Learning With Kernels for Portfolio Valuation and Risk Management, Swiss Finance Institute Research, Paper No. 19-34.

Martin Grothaus (University of Kaiserslautern, Germany)

An improved characterization theorem - its interpretation in terms of Mallivain calculus and applications to SPDEs

Abstract: We consider spaces of test and regular generalised functions of white noise. These spaces 20 years ago were characterized by holomorphy on infinite dimensional spaces together with an integrability condition. We, instead, give a characterisation in terms of U-functionals, i.e., classic holomorphic functions on the one dimensional field of complex numbers, together with the same integrability condition. The characterisation of regular generalised functions is useful for solving singular SPDEs. Whereas, the characterisation of test functions is useful for showing smoothness of solutions to SPDEs in the sense of Mallivain calculus. We present concrete examples confirming the usefulness in both cases.

Reference

M. Grothaus; J. Müller; A. Nonnenmacher (2021). An improved characterisation of regular generalised functions of white noise and an application to singular SPDEs. Accepted for publication in Stochastics and Partial Differential Equations: Analysis and Computations.

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Titles and Abstracts of Invited Speakers

Rajat Subhra Hazra (Indian Statistical Institute, Kolkata & Leiden University, Netherlands)

Scaling limit of some random interface models

Abstract: In this talk we will discuss two examples of interface models, namely the Gaussian free field and Membrane model. We will make a comparison between the two models and discuss some properties of the membrane model. We will discuss some results on the scaling limit of the Membrane model and some of its variants on the lattice. We will display that the techniques from PDE help us prove the scaling limit. This talk is based on joint works with Alessandra Cipriani (TU Delft) and Biltu Dan (IISc).

Reference

Alessandra Cipriani, Biltu Dan, Rajat Subhra Hazra. Scaling limit of semiflexible polymers: a phase transition. Communications in Mathematical Physics. 377, 1505-1544, 2020.

Alessandra Cipriani,Biltu Dan, Rajat Subhra Hazra. The scaling limit of the membrane model. Annals of Probability. Vol. 47, No. 6, 3963-4001, 2019.

Alessandra Cipriani, Biltu Dan, Rajat Subhra Hazra. The scaling limit of the $(\nabla+\Delta)$-model. Available at arXiv:1808.02676, 2020. To appear in the Journal of Statistical Physics.

Bernt Øksendal (University of Oslo, Norway)

SPDEs with space interactions - a model for optimal control of epidemics

Abstract: We consider optimal control of a new type of non-local stochastic partial differential equations (SPDEs). The SPDEs have space interactions, in the sense that the dynamics of the system at time t and position in space x also depend on the space-mean of values at neighbouring points. This is a model with many applications, e.g. to population growth studies and epidemiology. We prove the existence and uniqueness of solutions of such SPDEs with space interactions, and using white noise theory we show that, under some conditions, the solutions are positive for all times if the initial values are. Sufficient and necessary maximum principles for the optimal control of such systems are derived. Finally, we apply the results to study an optimal vaccine strategy problem for an epidemic by modelling the population density as a space-mean stochastic reaction-diffusion equation. The talk is based on joint work with Nacira Agram, Astrid Hilbert and Khouloud Makhlouf.

Szymon Peszat (Jagiellonian University Cracow, Poland)

Heat equations with white noise Dirichlet boundary conditions

Abstract: The talk in based on a joint work with Ben Goldys (University of Sydney). We study inhomogeneous Dirichlet boundary problems associated to heat equations on bounded and unbounded domains with white noise boundary data. We prove the existence of Markovian solutions living in weighted spaces of p-integrable functions where the weight is a proper power of the distance from the boundary.

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Titles and Abstracts of Invited Speakers

Michael Röckner (University of Bielefeld, Germany)

Nonlinear Fokker-Planck equations with measures as initial data and McKean-Vlasov equations

Abstract: This talk is about joint work with Viorel Barbu. We consider a class of nonlinear Fokker-Planck (-Kolmogorov) equations (FPEs) of type

$$\frac{\partial}{\partial t} u(t,x) - \Delta_x \beta(u(t,x)) + \operatorname{div}(D(x) b(u(t,x)) u(t,x)) = 0, \qquad u(0,\cdot)=\mu$$

where $(t,x) \in 0, [0,\infty) \times \mathbb{R}^d, d \geq 3$ and μ is a signed Borel measure on $\mathbb{R}^d$ of bounded variation.

We shall explain how to construct a solution to the above PDE based on classical nonlinear operator semigroup theory on $L^1(\mathbb{R}^d)$ and new results on $L^1 - L^\infty$ regularization of the solution semigroups in our case.

To motivate the study of such FPEs we shall first present a general result about the correspondence of nonlinear FPEs and McKean-Vlasov type SDEs. In particular, it is shown that if one can solve the nonlinear FPE, then one can always construct a weak solution to the corresponding McKean-Vlasov SDE. We would like to emphasize that this, in particular, applies to the singular case, where the coefficients depend “Nemytski-type” on the time-marginallaw law of the solution process, hence the coefficients are not continuous in the measure-variable with respect to the weak topology on probability measures. This is in contrast to the literature in which the latter is standardly assumed. Hence we can cover nonlinear FPEs as the ones above, which are PDEs for the marginal law densities, realizing an old vision of McKean. (pdf)

Reference

V. Barbu, M. Röckner: From nonlinear Fokker-Planck equations to solutions of distribution dependent SDE, Ann. Prob. 48 (2020), no. 4, 1902-1920.

V. Barbu, M. Röckner: Solutions for nonlinear Fokker-Planck equations with measures as initial data and McKean-Vlasov equations, J. Funct. Anal. 280 (2021), no. 7, 108926.

Sivaguru S. Sritharan (Applied Optimization, Inc., USA)

Navier-Stokes Equations: Ergodicity, Large Deviations, Control, Filtering and Malliavin Calculus

Abstract: In this talk we will give an overview of several rigorous studies and results over the past number of years on compressible and incompressible Navier-Stokes and Euler equations including martingale, pathwise and strong solutions, invariant measures and ergodicity, large deviations of small noise (Freidlin-Wentzell) and large time (Donsker-Varadhan) type, filtering, control and Malliavin calculus and related infinite dimensional partial differential equations, optimal stopping and impulse control and related infinite dimensional variational and quasi-variational inequalities. We will also outline developments in related subjects such as magnetohydrodynamics as well as PDEs in physics such as the Einstein field equation, Maxwell-Dirac equation and the nonlinear Schrodinger equation, all subject to Gaussian and Lévy noise.

Josef Teichmann (ETH Zurich, Switzerland)

Gaussian processes, Signatures and Kernelizations

Abstract: We shall introduce a functional analytic setting in the spirit of Röckner-Sobol for signatures on path space, where kernelization techniques can then be applied. This allows for efficient training of approximations of path space functionals given through data. Applications from Finance are shown. Joint work with Christa Cuchiero and Philipp Schmocker.

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