Nonlinear Fokker-Planck equations with measures as initial data and McKean-Vlasov equations

This talk is about **joint work with Viorel Barbu**. We consider a class of nonlinear Fokker-Planck (-Kolmogorov) equations (FPEs) of type

$$\frac{\partial}{\partial t}u(t,x) - \Delta_x \beta(u(t,x)) + \operatorname{div}\big(D(x)b(u(t,x))u(t,x)\big) = 0, \quad u(0,\cdot) = \mu,$$

where $(t,x) \in [0,\infty) \times \mathbb{R}^d$, $d \geq 3$ and μ is a signed Borel measure on $\mathbb{R}^d$ of bounded variation.

We shall explain how to construct a solution to the above PDE based on classical nonlinear operator semigroup theory on $L^1(\mathbb{R}^d)$ and new results on $L^1 - L^\infty$ regularization of the solution semigroups in our case.

To motivate the study of such FPEs we shall first present a general result about the correspondence of nonlinear FPEs and McKean-Vlasov type SDEs. In particular, it is shown that if one can solve the nonlinear FPE, then one can always construct a weak solution to the corresponding McKean-Vlasov SDE. We would like to emphasize that this, in particular, applies to the singular case, where the coefficients depend “Nemytski-type” on the time-marginal law of the solution process, hence the coefficients are not continuous in the measure-variable with respect to the weak topology on probability measures. This is in contrast to the literature in which the latter is standardly assumed. Hence we can cover nonlinear FPEs as the ones above, which are PDEs for the marginal law densities, realizing an old vision of McKean.

References

V. Barbu, M. Röckner: *From nonlinear Fokker-Planck equations to solutions of distribution dependent SDE*, Ann. Prob. **48** (2020), no. 4, 1902-1920.

V. Barbu, M. Röckner: *Solutions for nonlinear Fokker-Planck equations with measures as initial data and McKean-Vlasov equations*, J. Funct. Anal. **280** (2021), no. 7, 108926.